ITERATIVE APPLICATION OF THE FORCE DENSITY METHOD

Elizabeta Šamec, esamec@grad.hr
Krešimir Fresl
Maja Baniček
University of Zagreb, Faculty of Civil Engineering,
Zagreb, Croatia

Faculty of Civil Engineering, University of Zagreb, Croatia
Department of technical mechanics
Chair for statics, dynamics and stability of structures
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Introduction

- **OBJECTIVE**:
  - to improve the design of long span roofs according to the geometric and structural criteria

Cable net structures

- structural shape and values of prestressed forces are most important – geometric stiffnes
- form finding – shape that satisfies structural and architectural requirements
- inverse problem – determination of initial equilibrium configuration
- nonlinear equations of mathematical model

- **IDEA** is to speed up the procedure of designing cable structures with iterative application of the force density method
  - to apply the force density method (linearization) in each step
  - to determinate force densities in a given step with regard to specific requirements and results of preceding step
  - reduce time required for calculation as well as number of steps

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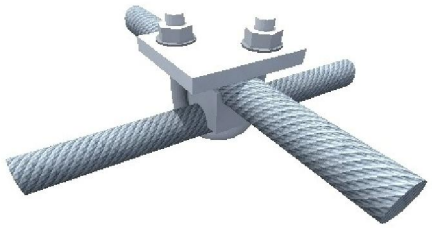
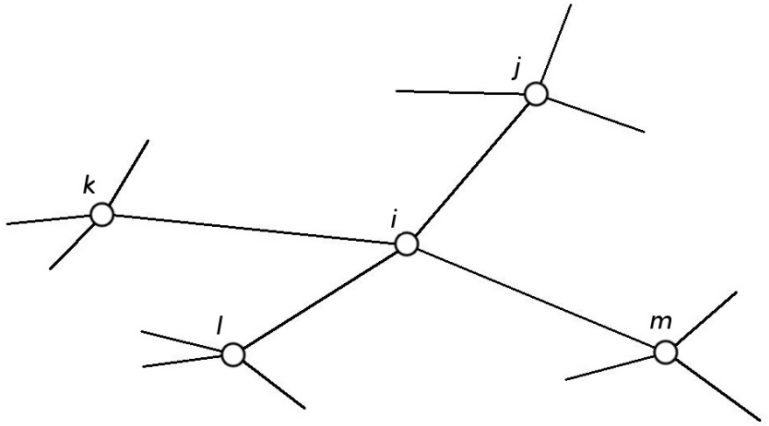
Computational model



- cable segments between the crosing points are hinged bar elements centrically attached to nodes (free and fixed)
  - ➢ space truss



Form finding

- defining initial equilibrium configuration – without external loads (no self weight)
- **variables** :
  - shape

    defined by node coordinates - basic unknowns
  - topology

    determined by assumed layout and number of cables
  - geometric boundary conditions

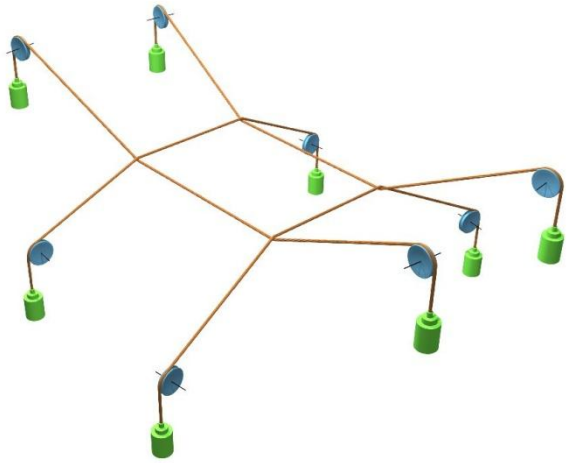
    determined by given coordinates of fixed nodes
  - values of prestressing forces

    given or determined by the condition of nodal equilibrium

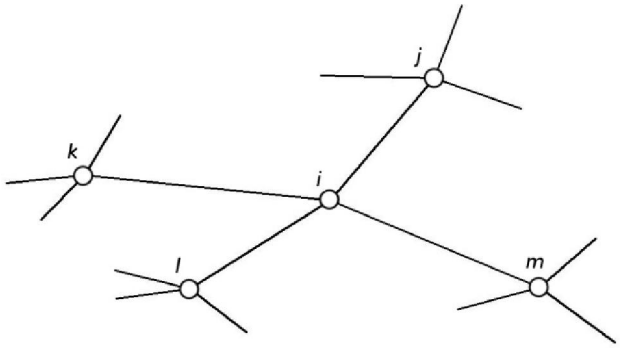
    (additional kinematic constrains)

    by changing the ratio of force values we can cange the shape of the net

Form finding



- starting point – equilibrium equations of free nodes that are acted on only by prestressing forces in connected bars
- for each free node *i* three equilibrium equations can be written
- if *b* is the number of cables and n is the number of free nodes, we have 3*n* + *b* unknowns



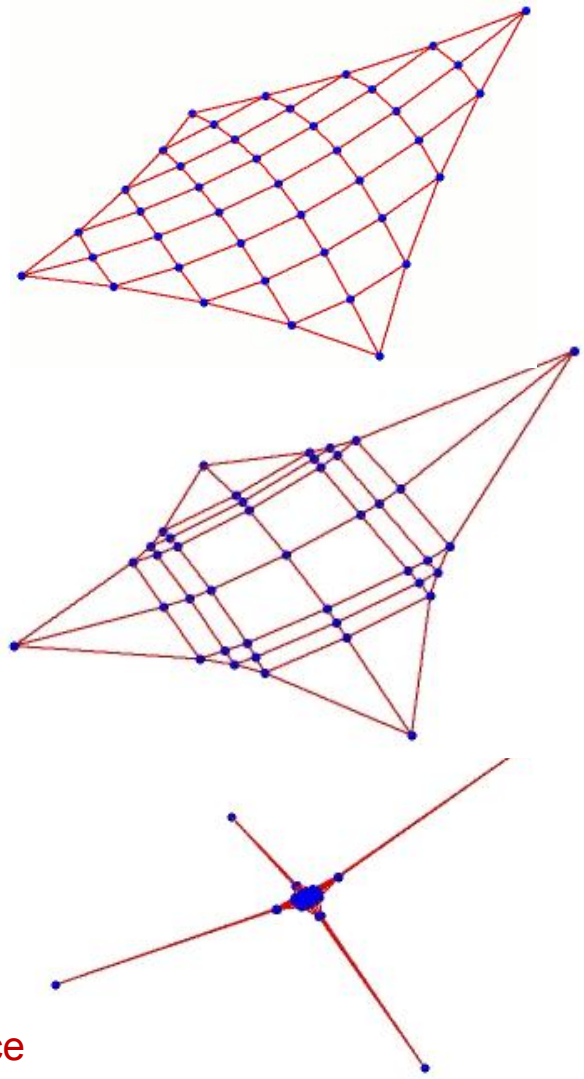
$$\sum_j S_{i,j} \frac{x_j - x_i}{\ell_{i,j}} = 0$$

$$\sum_j S_{i,j} \frac{x_j - x_i}{\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}} = 0$$

Generalized minimal nets

- equilibrium conditions can be interpreted as the condition for the minimum of the function

$$\mathcal{E}\left(\{x_k, y_k, z_k\}_{k\in\mathcal{N}}\right) = \sum\nolimits_{\{i,j\}\in\mathcal{B}} S_{i,j}\,\ell_{i,j}$$



Sliding problem

- sliding of cables must not be prevented to enable minimal configuration
- regardless of the force values, nodes can slide into a single point – nontrivial nonsingular equilibrium state does not exist
- solution: to specify bar lengths (forces are unknown)
  - ➢ kinematic constrains (using Lagrange multipliers)
  - ➢ duration of calculation is longer and convergence domain is smaller

Force density method

- by specifing the force-to-lenght ratio for each element, equilibrium equations are linearized

$$q_{i,j} = S_{i,j}/\ell_{i,j}$$

$$\sum_j q_{i,j}\,(x_j - x_i) = 0$$

- by defining force densities in elemets, the system of 3*n* equilibrium equations is decomposed into three intependant systems (in coordinate directions) with same system matrix
- as a result we obtain coordinates of free nodes and we can calculate the lenghts and the forces in elements
- any shape obtained by using FDM is equilibrium configuration
- the problem is how to select force density values in order to obtain the desirable shape (structural and architectural requirements)

Iterative application of force density method

- iterative FDM enables gaining determinated values of either lenghts or forces in elements
- force densities in a given step are detemined with regard to specific requirements and results of preceding step
- required force values (bar lengths) can be attained by computing force densities in k-th iteration step according to expressions:

$$q_{i,j}^{(k)} = q_{i,j}^{(k-1)} \frac{\bar{S}_{i,j}}{S_{i,j}^{(k-1)}}, \quad q_{i,j}^{(k)} = \frac{\bar{S}_{i,j}}{\ell_{i,j}^{(k-1)}}$$

$$q_{i,j}^{(k)} = q_{i,j}^{(k-1)} \frac{\ell_{i,j}^{(k-1)}}{\bar{\ell}_{i,j}}, \quad q_{i,j}^{(k)} = \frac{S_{i,j}^{(k-1)}}{\bar{\ell}_{i,j}}$$

- Iteration is terminated when :

$$r_S^{(k)} = \max_{\{i,j\}}(|S_{i,j}^{(k)} - \bar{S}_{i,j}|) < \tau_S$$

$$r_\ell^{(k)} = \max_{\{i,j\}}(|\ell_{i,j}^{(k)} - \bar{\ell}_{i,j}|) < \tau_\ell$$

Iterative application of force density method

- ADVANTAGE: iteration procedure converges through series of equilibrium configurations toward solution – procedure can be stoped at any time
- three systems of linear equations are solved in each step (same system matrix)

Solution methods

- **LU decomposition -** sistem with more right sides can be solved

- **Conjugate gradient method** - system matrices are symmetric and positive–definite
  - three systems must be separately solved
  - application of inexact Newton methods idea

Appliaction of inexact Newton methods idea

- the main idea is to solve linear systems only approximately (when values of forces and lenghts are far from the searched ones)
- provides a compromise between the accuracy of the linear systems solution and the amount of work per iteration step
- advantage of the iterative application of FDM is sacrificed – intermediate configurations are not balanced since linarized equations in earlier steps are solved with smaller accuracy

The research is concentrated on a choice of the termination rule that will prevent the accuracy of linear solutions from to quickly becoming unnecessarily high, at the same time retaining the convergence of the iterative force density method.

- accuracy in each step is gradually increasing toward specified accuracy τ_e of the final solution of equilibrium equations
- accuracy in k-th step can not be smaller then accuracy of previous step:

$$\tau^{(k)} = \min\left(\tau^{(k-1)}, \tau_r^{(k)}\right), \qquad \tau^{(0)} = \tau_{\max}$$

$$\tau_r^{(k)} = \max\left(\tau_{rS}^{(k)}, \tau_{r\ell}^{(k)}, \tau_e\right)$$

$$\tau_{rS}^{(k)} = \min\left(\gamma_S \left(r_S^{(k-1)}\right)^2, \eta \frac{\left(r_S^{(k-1)}\right)^3}{\left(r_S^{(k-2)}\right)^2}\right), \qquad \tau_{r\ell}^{(k)} = \min\left(\gamma_\ell \left(r_\ell^{(k-1)}\right)^2, \eta \frac{\left(r_\ell^{(k-1)}\right)^3}{\left(r_\ell^{(k-2)}\right)^2}\right)$$

$$\gamma_S = \frac{\tau_e\left(1-\sqrt{\tau_S}\right)}{\tau_S^2}, \qquad \gamma_\ell = \frac{\tau_e\left(1-\sqrt{\tau_\ell}\right)}{\tau_\ell^2}$$

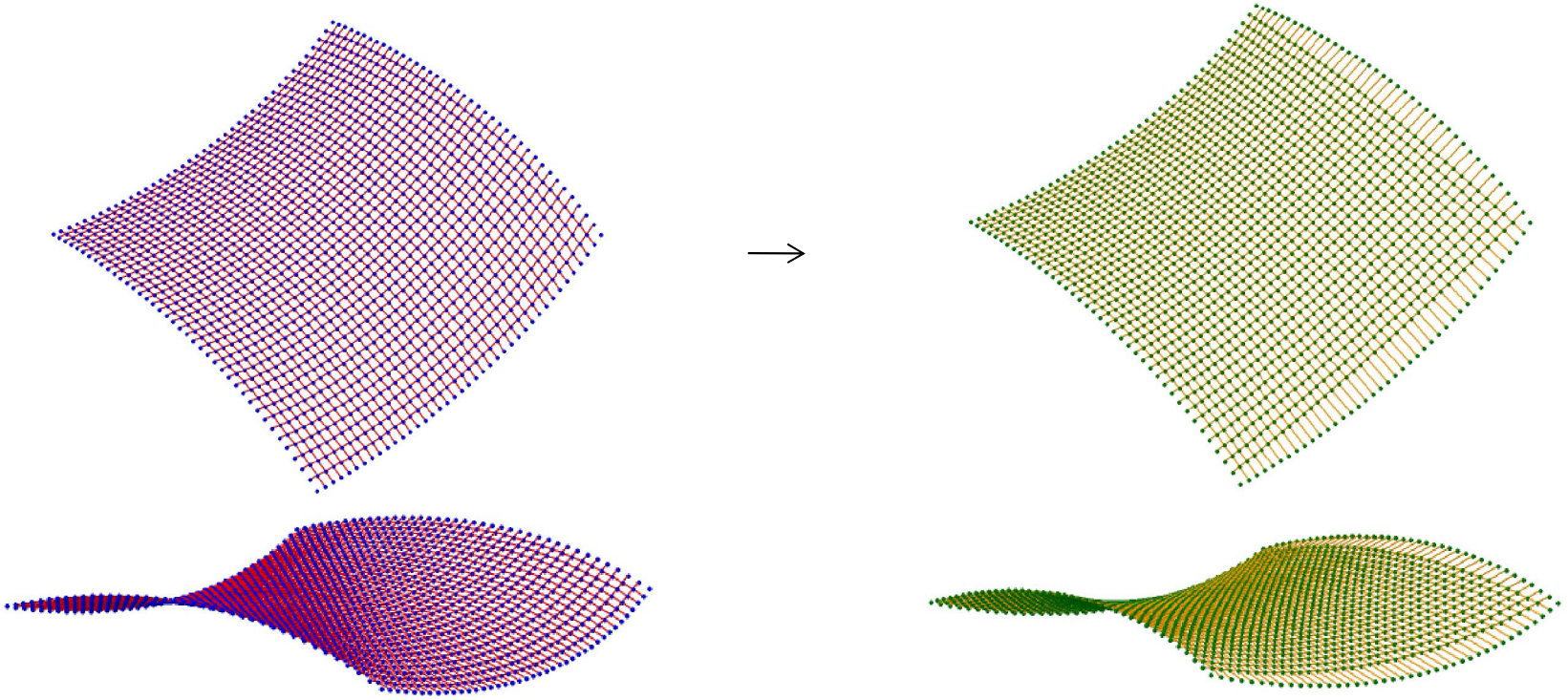
Example 1

1600 nodes
1444 free nodes $\longrightarrow$ 4332 equations
2964 elements

$\tau_s = 10^{-4}$

$\tau_e = 5 \cdot 10^{-7}$

	Conjugate gradient method		Inexact CGM
	$\mathbb{X}_0^{(k)} = 0$	$\mathbb{X}_0^{(k)} = \mathbb{X}^{(k-1)}$	
Outer steps	774	774	677
Inner steps	233 108	60 275	9 170



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Example 2

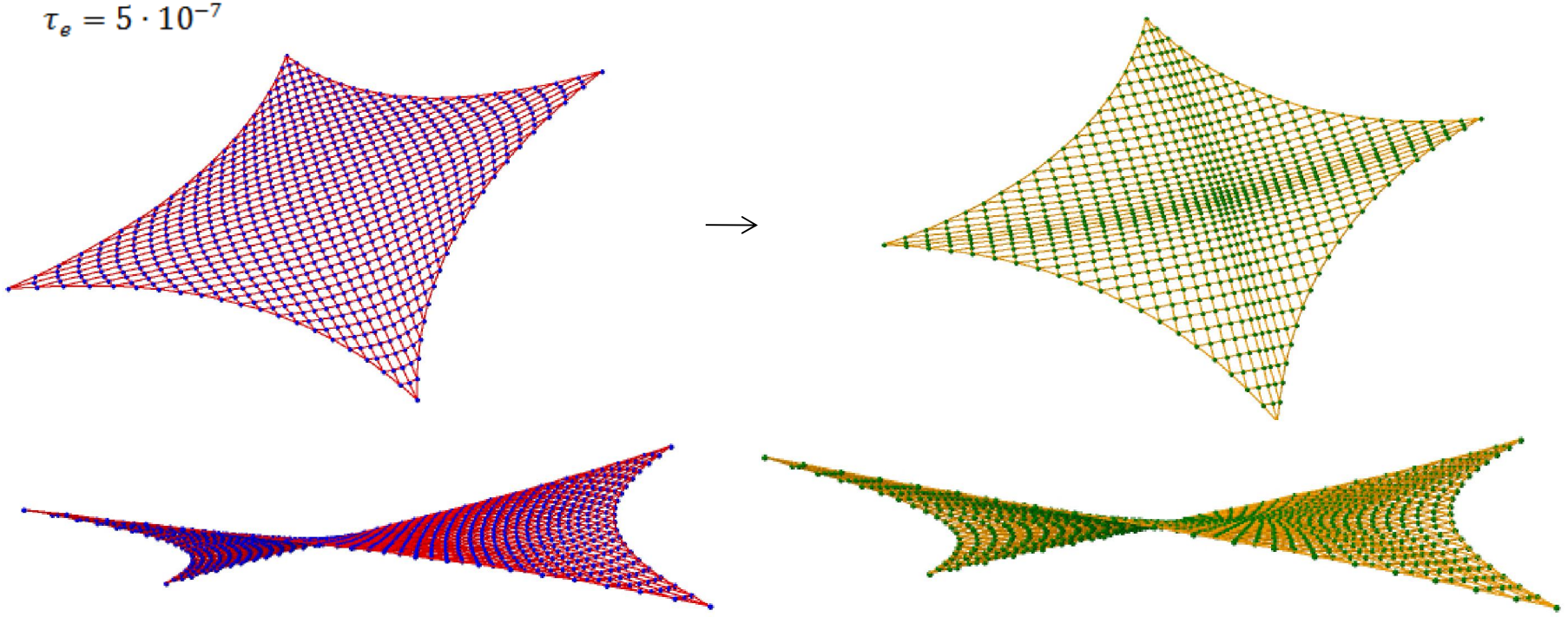
841 nodes
837 free nodes $\longrightarrow$ 2511 equations
1680 elements

$\tau_s = 10^{-4}$

$\tau_l = 10^{-4}$

$\tau_e = 5 \cdot 10^{-7}$

	Conjugate gradient method		Inexact CGM
	$\mathbb{X}_0^{(k)} = 0$	$\mathbb{X}_0^{(k)} = \mathbb{X}^{(k-1)}$	
Outer steps	871	949	950
Inner steps	571 859	45 606	16 562



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Example 3

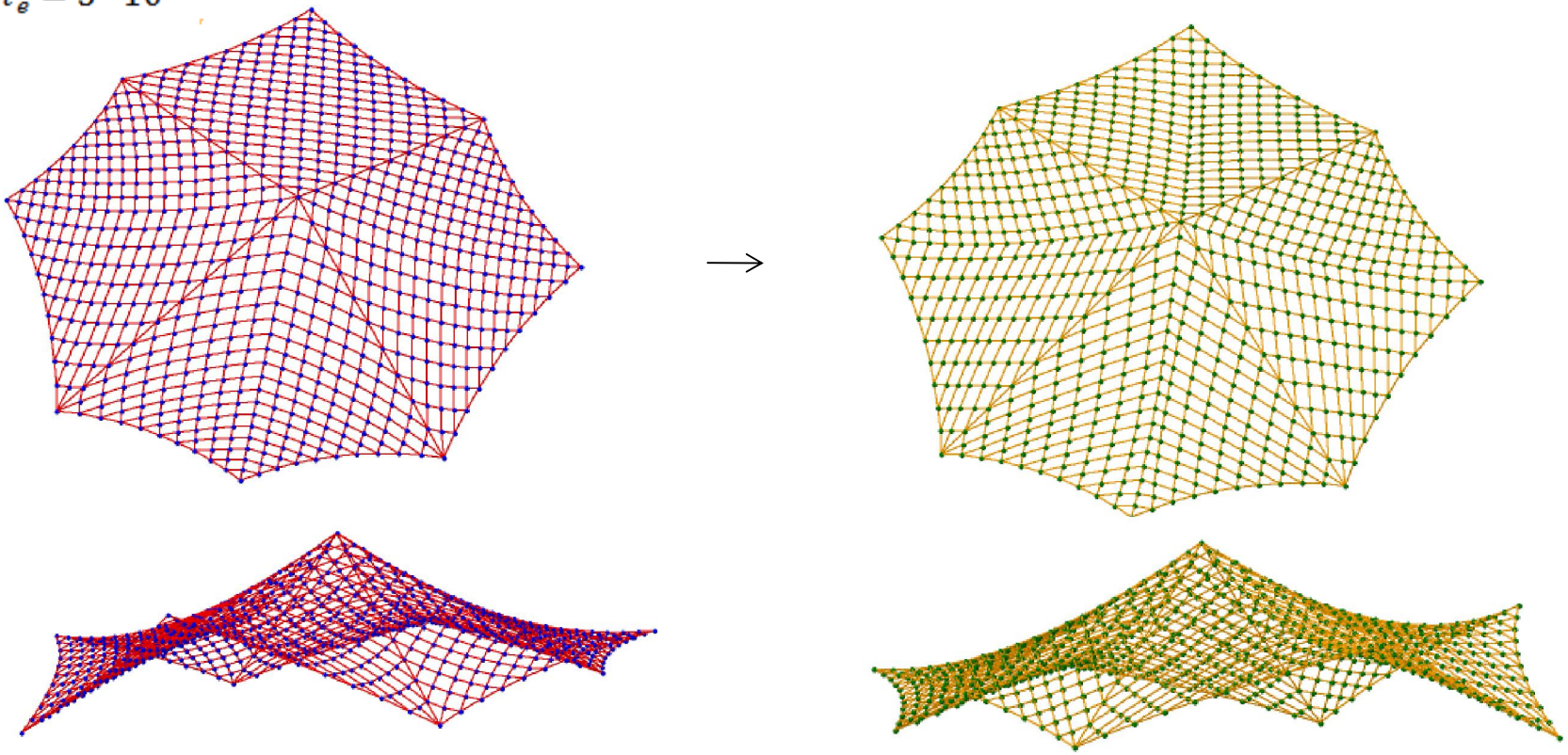
841 nodes
832 free nodes ⟶ 2496 equations
1720 elements

$\tau_s = 10^{-4}$

$\tau_l = 10^{-3}$

$\tau_e = 5 \cdot 10^{-7}$

	Conjugate gradient method		Inexact CGM
	$\mathbb{X}_0^{(k)} = 0$	$\mathbb{X}_0^{(k)} = \mathbb{X}^{(k-1)}$	
Outer steps	299	300	301
Inner steps	146 385	21 001	7 287



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Conclusion

- With iterative application of force density method we can obtain specified values of lenghts or forces in elements, but in some cases problem can be the duration of calculations.

- Extensive numerical experiments show that the proposed method is almost always efficient and robust, although there are cases in which the efficiency strongly depends on constants in proposed termination rule.

- We belive that proposed idea of “speeding up” the computation of cable structures will enable integration of FDM in to interactive environment for form finding.

Thank You

Elizabeta Šamec, esamec@grad.hr

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